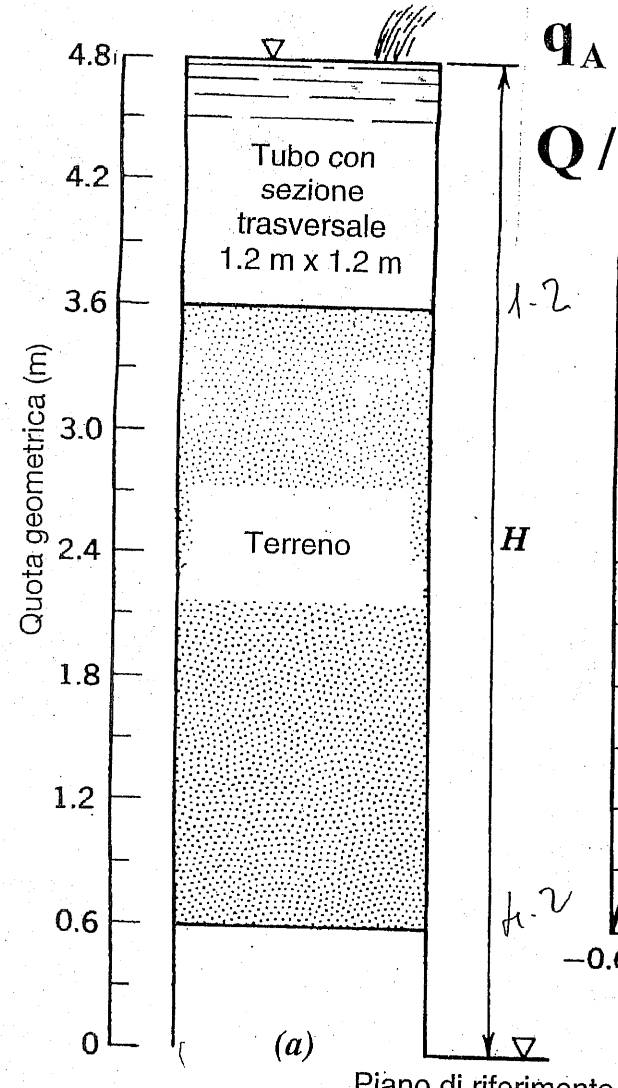
IDRAULICA: FLUSSO STAZIONARIO

MOTO DI FILTRAZIONE MONODIMENSIONALE

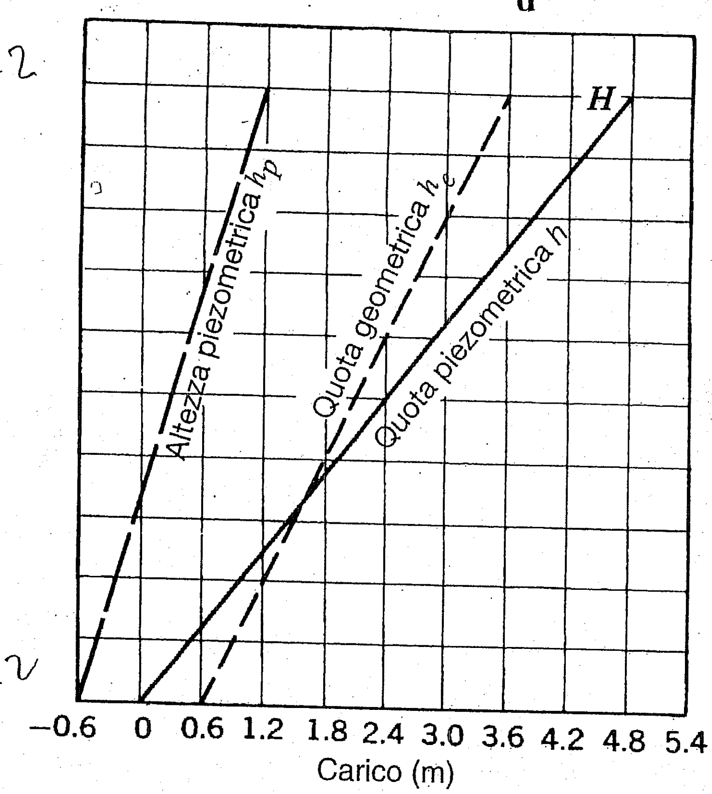
$Q = kiA$ $\quad (k = 0.05 \cdot 10^{-2}\, m/s) \quad A = 1.2 \times 1.2$

$q_A = k \cdot i_A \cdot a_A = k \cdot H \cdot b / n_d l$ (portata per L=1.0 m)

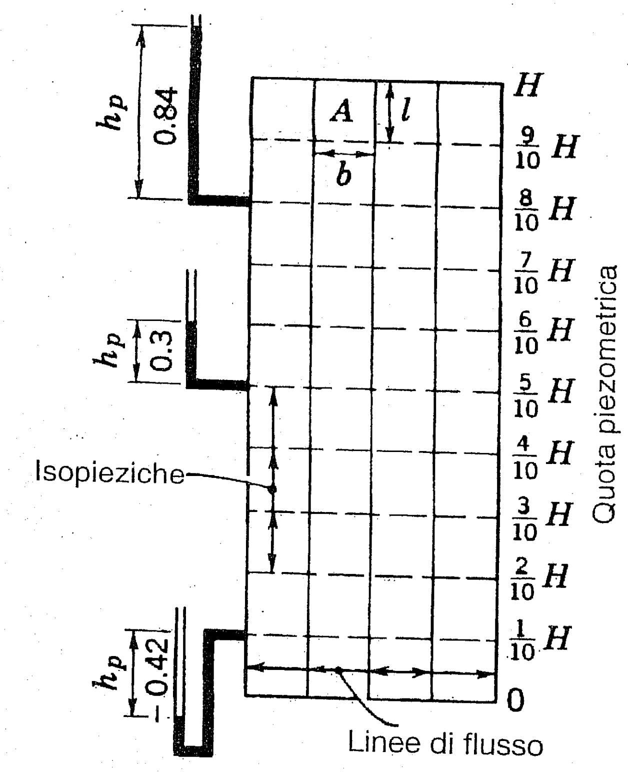
$Q/L = q_A \cdot n_f = k \dfrac{n_f}{n_d} H \qquad FF = n_f / n_d$



(a)

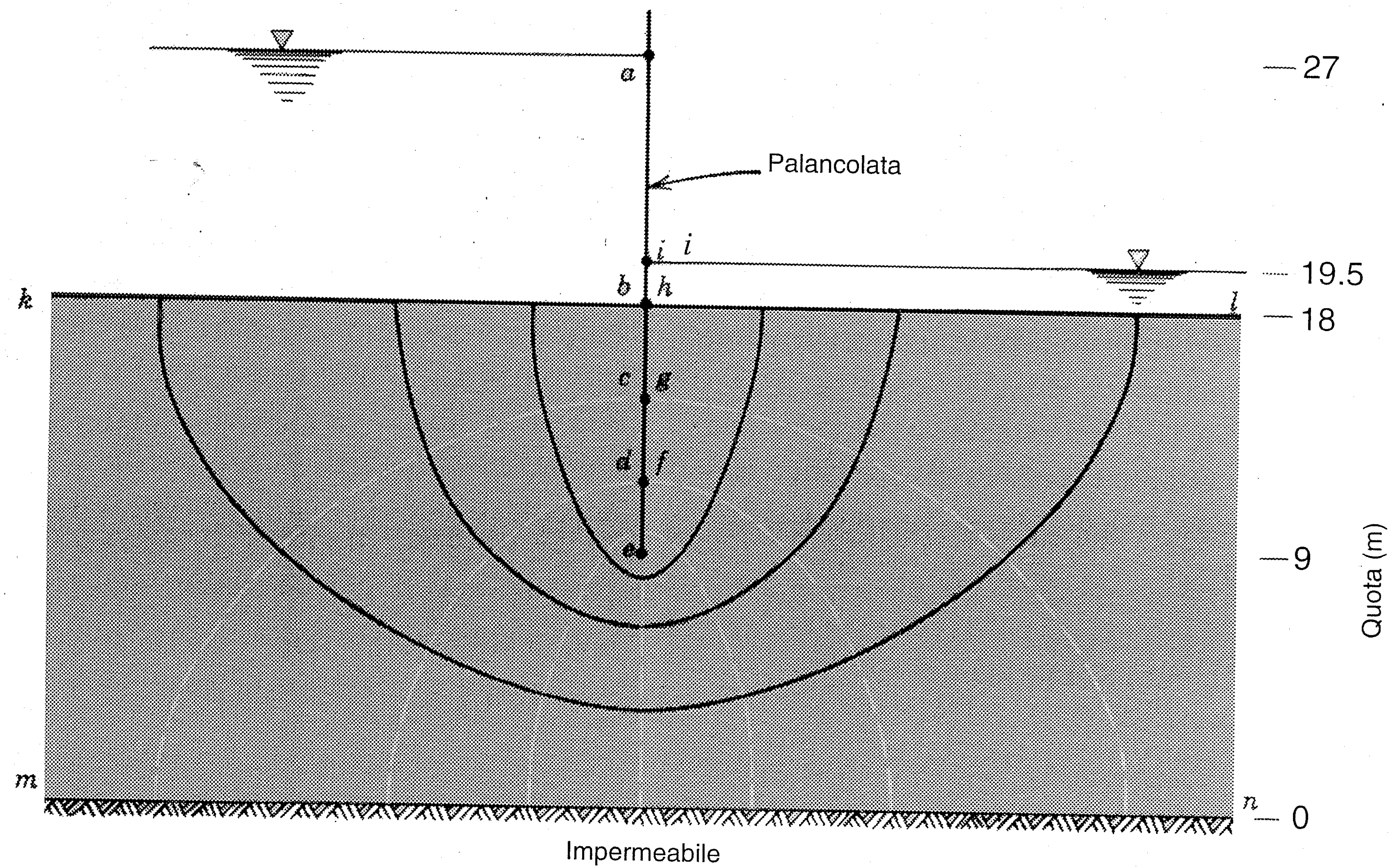


(b)

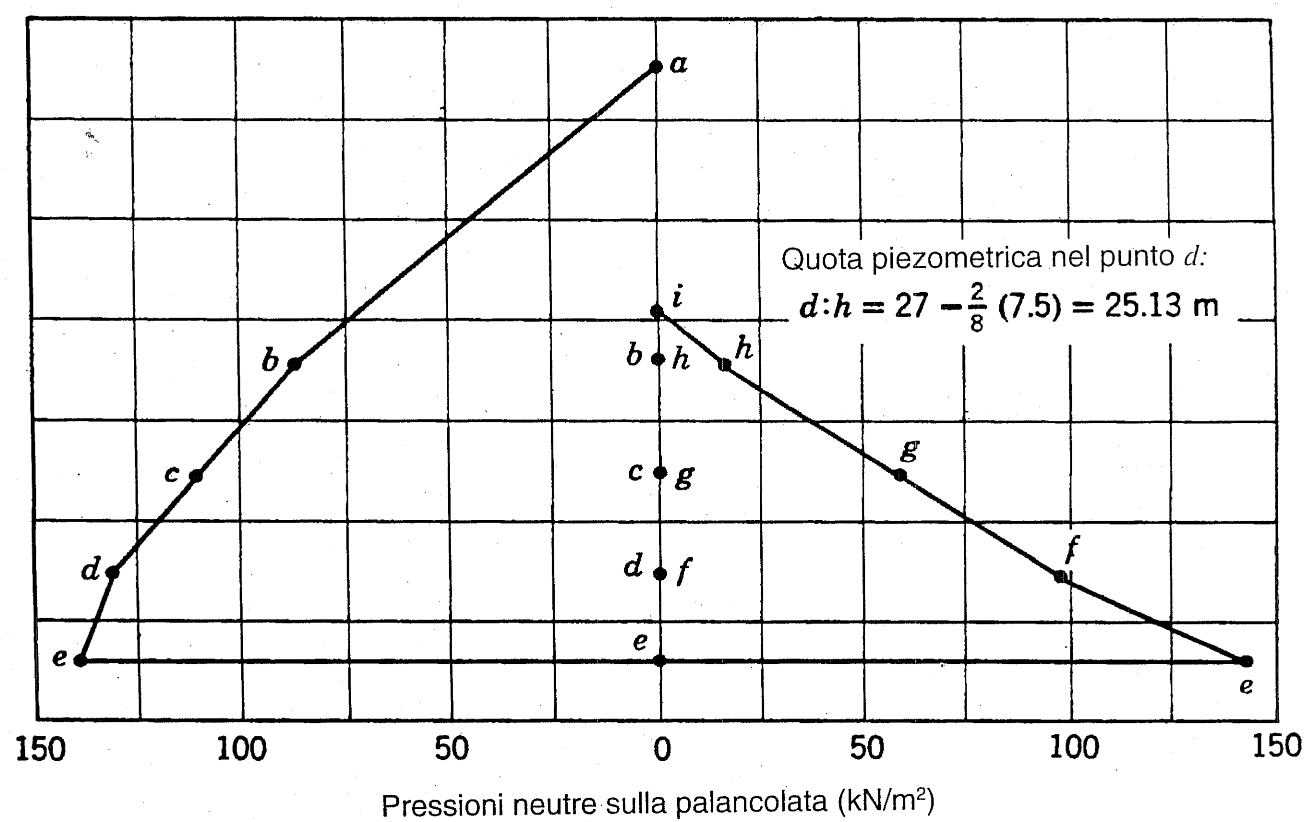


(c)

MOTO DI FILTRAZIONE PIANO



MOTO DI FILTRAZIONE PIANO



EQUAZIONE DI FLUSSO (1)

IPOTESI:
- Interazione tra le fasi (sforzi efficaci)
- Continuità (conservazione della massa)
- Equazione di stato (densità del fluido)
- Equilibrio dinamico (Darcy)

$$q = q_x + q_y + q_z$$

q_z

IN $\quad K_z \cdot \left(-\frac{\partial h}{\partial z} \right) \cdot dx \cdot dy$

OUT $\quad \left(K_z + \frac{\partial K_z}{\partial z} \cdot dz \right) \left(-\frac{\partial h}{\partial z} - \frac{\partial^2 h}{\partial z^2} dz \right) \cdot dx \cdot dy$

Massa $(\rho_w \cdot v_i)$

equilibrio $(v_i = K_i \cdot \frac{\partial h}{\partial i})$

permeabilità costante $(K_z = \text{costante})$

$$\Delta q_z = \left(K_z \frac{\partial^2 h}{\partial z^2} \right) \cdot dx \cdot dy \cdot dz$$

EQUAZIONE DI FLUSSO (2)

Volume d'acqua

$$V_w = \frac{S \cdot e}{1+e} dx \cdot dy \cdot dz$$

$$\Delta q = \frac{\partial V_w}{\partial t} = \frac{\partial}{\partial t}\left(\frac{S \cdot e}{1+e} dx \cdot dy \cdot dz\right)$$

$$\Delta q = \frac{dx \cdot dy \cdot dz}{1+e} \frac{\partial S \cdot e}{\partial t}$$

$$\left(K_z \frac{\partial^2 h}{\partial z^2} + K_x \frac{\partial^2 h}{\partial x^2}\right) \cdot dx \cdot dy \cdot dz = \frac{dx \cdot dy \cdot dz}{1+e} \frac{\partial S \cdot e}{\partial t}$$

$$K_z \frac{\partial^2 h}{\partial z^2} + K_x \frac{\partial^2 h}{\partial x^2} = \frac{1}{1+e}\left(e \frac{\partial S}{\partial t} + S \frac{\partial e}{\partial t}\right)$$

a) e, S costanti — **filtrazione**
b) e varia S costante — **consolidazione**
c) e costante S varia — **drenaggio/imbibizione**
d) e, S variano

$$K_z \frac{\partial^2 h}{\partial z^2} + K_x \frac{\partial^2 h}{\partial x^2} = 0$$

$$\frac{\partial^2 h}{\partial z^2} + \frac{\partial^2 h}{\partial x^2} = 0$$

SOLUZIONE DELL'EQUAZIONE DI LAPLACE

Sviluppo in serie di Taylor

$$f(x) = f(x_o) + f'(x_o)\cdot(x - x_o) + \frac{f''(x_o)}{2!}\cdot(x - x_o)^2 + \frac{f'''(x_o)}{3!}\cdot(x - x_o)^3$$

$$h_1 = h_o + \left(\frac{\partial h}{\partial x}\right)_o \cdot \Delta x + \left(\frac{\partial^2 h}{\partial x^2}\right)_o \cdot \frac{\Delta x^2}{2!} + \left(\frac{\partial^3 h}{\partial x^3}\right)_o \cdot \frac{\Delta x^3}{3!} +$$

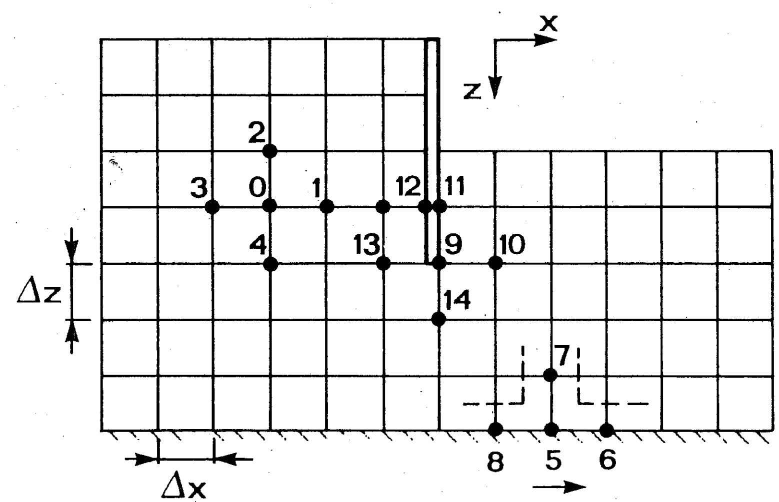
$$h_3 = h_o - \left(\frac{\partial h}{\partial x}\right)_o \cdot \Delta x + \left(\frac{\partial^2 h}{\partial x^2}\right)_o \cdot \frac{\Delta x^2}{2!} - \left(\frac{\partial^3 h}{\partial x^3}\right)_o \cdot \frac{\Delta x^3}{3!} +$$

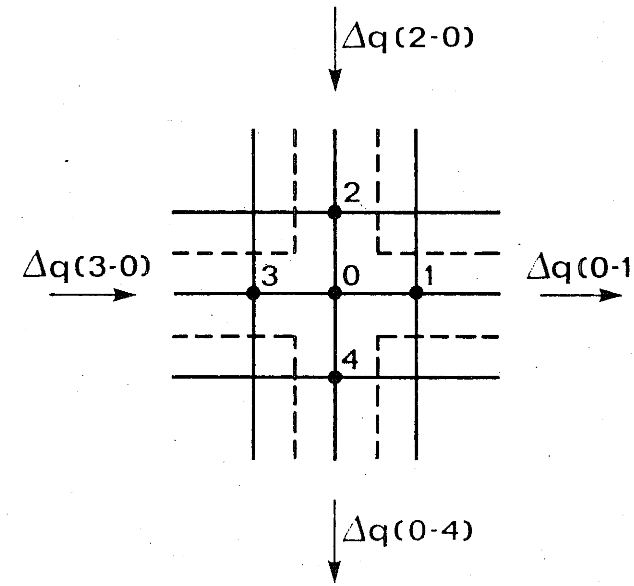
$$\left(\frac{\partial^2 h}{\partial x^2}\right)_o = \frac{h_1 + h_3 - 2h_o}{\Delta x^2}$$

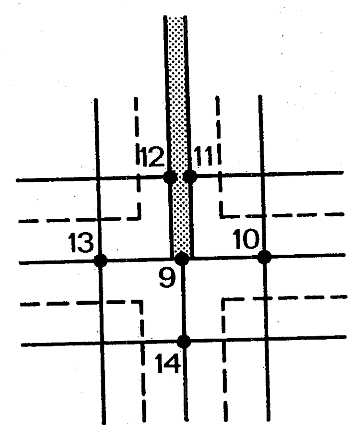
$$\left(\frac{\partial^2 h}{\partial z^2}\right)_o = \frac{h_2 + h_4 - 2h_o}{\Delta z^2}$$

$$\frac{h_1 + h_3 - 2h_o}{\Delta x^2} + \frac{h_2 + h_4 - 2h_o}{\Delta z^2} = 0$$

RELAZIONE DI CONTINUITA'







RELAZIONE DI CONTINUITA'

$$\Delta q(2-0) = K_z i \Delta x = K_z \frac{h_2 - h_o}{\Delta z} \Delta x$$

$$\Delta q(0-4) = K_z i \Delta x = K_z \frac{h_o - h_4}{\Delta z} \Delta x$$

$$\Delta q(3-0) = K_x i \Delta z = K_x \frac{h_3 - h_o}{\Delta x} \Delta z$$

$$\Delta q(0-1) = K_x i \Delta z = K_x \frac{h_o - h_1}{\Delta x} \Delta z$$

$$\left[\frac{h_8 - h_5}{\Delta x} \frac{\Delta z}{2}\right] - \left[\frac{h_5 - h_6}{\Delta x} \frac{\Delta z}{2}\right] - \left[\frac{h_5 - h_7}{\Delta z} \Delta x\right] = 0$$

$$\left[\frac{h_8}{2}\right] + \left[\frac{h_6}{2}\right] + h_7 - 2h_5 = 0$$

$$(h_{13} - h_9) - (h_9 - h_{10}) + (h_{14} - h_9) -$$

$$0.5(h_9 - h_{12}) - 0.5(h_9 - h_{11}) = 0$$

MEZZO ETEROGENEO O ANISOTROPO

$$\frac{K_x}{\Delta x^2}(h_1 + h_3 - 2h_o) + \frac{K_z}{\Delta z^2}(h_2 + h_4 - 2h_o) = 0$$

$$\Delta x = \Delta z \cdot (K_x / K_z)^{0.5}$$

oppure

$$K_x \cdot \frac{\partial^2 h}{\partial x^2} + K_z \cdot \frac{\partial^2 h}{\partial z^2} = 0$$

$$y = x(K_z / K_x)^{0.5}$$

$$q = K_e i \Delta z = K_e \frac{\Delta h}{\Delta y} \Delta z$$

$$q = K_x \frac{\Delta h}{\Delta x} \Delta z = K_x \frac{\Delta h}{\Delta y (K_x / K_z)^{0.5}} \Delta z = (K_x K_z)^{0.5} \Delta h$$

MOTO NON CONFINATO

IPOTESI DI DUPUIT:

- gradiente idraulico costante in un piano V
- gradiente idraulico=pendenza
- linee di flusso orizzontali

$$q_x = K_x \cdot \left(-\frac{\partial h}{\partial x} \right) \cdot h \cdot dy$$

$$q_{x+dx} = K_x \cdot \left(-\frac{\partial h}{\partial x} \right) \cdot h \cdot dy + K_x \cdot \frac{\partial}{\partial x} \left(-h \cdot \frac{\partial h}{\partial x} \right) \cdot dx \cdot dy$$

$$\Delta q_x = K_x \cdot \frac{\partial}{\partial x} \left(h \frac{\partial h}{\partial x} \right) \cdot dx \cdot dy$$

MOTO STAZIONARIO

$$K_y \frac{\partial^2 h^2}{\partial y^2} + K_x \frac{\partial^2 h^2}{\partial x^2} = 0$$

$$\frac{\partial^2 h^2}{\partial y^2} + \frac{\partial^2 h^2}{\partial x^2} = 0$$